Novel method for determining the center of gravity height of mass standards

Neuartige Methode zur Bestimmung der Schwerpunkhöhe von Massenormalen

Markus Pabst und Thomas Fröhlich
Technische Universität Ilmenau, Ilmenau, Germany, markus.pabst@tu-ilmenau.de

Kurzfassung

Die genauesten Vakuummassekomparatoren haben eine Auflösung von 0.1 µg bei einem Kilogramm [1], entsprechend einer relativen Differenz der Masseartefakte von 1×10^{-10}. Schon kleine Unterschiede in der Höhe der Schwerpunkte der zu vergleichenden Masseartefakte führen zu einer Beeinflussung der Messergebnisse. Im Schwerefeld der Erde verursacht eine Schwerpunkthöhendifferenz von 1 cm einen Beitrag zur relativen Messunsicherheit von 3.2×10^{-9}. Daher ist es wichtig die genauen Schwerpunkthöhen der Artefakte zu kennen, sowie sich diese unterscheiden. Im Rahmen dieser Arbeit wird eine neuartige Methode vorgestellt, die es ermöglicht die Schwerpunkthöhen von hoch empfindlichen Masseartefakten zu bestimmen. Die Methode ermöglicht den Vergleich von Artefakten mit ungewöhnlicher Form, inhomogener Zusammensetzung oder Hohlmassenormalen mit unsicherer Innengeometrie. Dazu werden die Masseartefakte auf einem Präzisionskipptisch [2] auf drei 120° zueinander versetzten Kraftmesszellen abgesetzt. Durch die Kraftänderung bei Veränderung der Winkellage durch den Kipptisch um zwei Achsen, kann daraus resultierend die Schwerpunkthöhe ermittelt werden. Unterschiedliche Massenormale mit bekannter Geometrie und Zusammensetzung durchlaufen diesen Prozess zur Verifizierung der Methode. Letztlich zeigt der Vergleich zwischen theoretisch ermittelten Werten und den Messergebnissen eine gute Übereinstimmung der Methode.

Abstract

The most precise vacuum mass comparators have a resolution of 0.1 µg, operating at one kilogram [1]. The resolution represents a relative difference in weight of the artifacts of 1×10^{-10}. Small deviations in the height of the center of gravity of the mass artifacts lead to a systematic error of the measurement results. For example a difference in the height of the center of gravity of 1 cm in Earth's gravitational field causes a relative systematic uncertainty contribution of 3.2×10^{-9}. Therefore, it is important to know the artifacts' heights of the center of gravity. This paper introduces a novel approach to measure the height of the center of gravity of high-sensitive mass artifacts. The method allows to compare the mass of artifacts of unusual shape, inhomogeneous composition or hollow mass standards with uncertain internal geometry. For the determination, the mass artifacts are placed on three load cells, arranged at an angle of 120°. The load cells are attached to the base, which stands on a high-precision tilt table [2]. The difference in force with variation of the inclination angles allows to calculate the height of the center of gravity of the artifacts. To verify the method, various mass standards of known geometry and composition are measured as described. In the end, the comparison between theoretically determined values and the measurement results shows a good agreement of the method.

1 Introduction

For mass determinations in the highest range of accuracy mass comparators with electromagnetic force compensated (EMFC) weighing cells are state of the art. The base of a mass comparator is placed on a large weighing stone to prevent tilts and vibrations. Temperature variations affect the magnetic properties of the magnet system as well as magnetic materials surrounding construction. Fluctuations of the local gravitational field ([3]) normally have no influence on the measurement results, since this is abbreviated by the counterweight principle. Measurements are usually performed under high vacuum conditions to avoid measurement uncertainties due to water layers on the mass samples or buoyancy. In particular, buoyancy must be taken into account when mass samples of different materials are used. The International Prototype Kilogram ([4]) (also known as IPK or $\mathfrak{K}$) is made of platinum-iridium (Pt-Ir), while most industrial standards are made of stainless steel. As part of the redefinition of the International System of Units (SI) in 2019 [5] and the Avogadro project, a sphere was made of silicon-28 for mass comparisons. In the comparisons the height of the center of gravity of the mass samples differ due to the shape of the mass sample. The center of gravity of cylindrical or spherical mass standards with homogeneous composition can be easily cal-

ISBN 978-3-8007-5835-7

culated geometrically. In the future, mass artifacts with irregular shape, inhomogeneous composition, or uncertain interior geometry are also conceivable. For these types of artifacts, geometric calculation of the center of gravity is not possible. In this case, it is necessary to gently determine the center of gravity of the mass sample. In this paper, a device for measuring the center of gravity of mass samples up to a weight of 3 kg is presented.

2 State of the art

Knowing the center of gravity is important for many things. For example, the center of gravity of cars [6], especially high performance vehicles, is important for their stability on the road. Other examples are trucks [7] or wheelchairs [8]. Especially in the automotive industry, many methods for measuring the center of gravity have been established. The measurement method presented in this paper for measuring the center of gravity involves highly sensitive mass standards. Dynamic methods are not useful especially for hollow mass standards.

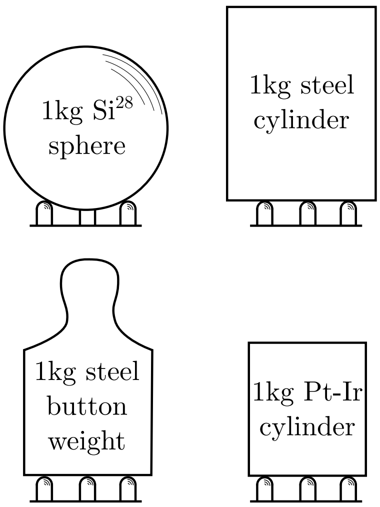

Figure 1 Different types of mass standards.

The most accurate vacuum mass comparators have a resolution of 0.1 µg for comparisons of 1 kg [1], corresponding to a detectable relative measurement error of 1×10^{-10} between mass artifacts. In the Earth's gravity field, a difference in the height of the center of gravity of 1 cm between two mass samples causes a contribution to the relative measurement uncertainty of 3.2×10^{-9} when compared.
As described in Section 1, mass samples for comparisons in mass comparators may have different shapes due to the material (and density) from which they are made. In Figure 1, known conventional mass samples are presented (see also [9]). Because of the use of the silicon sphere, only mass comparators with three-point support are considered.

3 Theory

The intended goals in measuring the center of gravity are a quick and easy determination as well as gentle handling

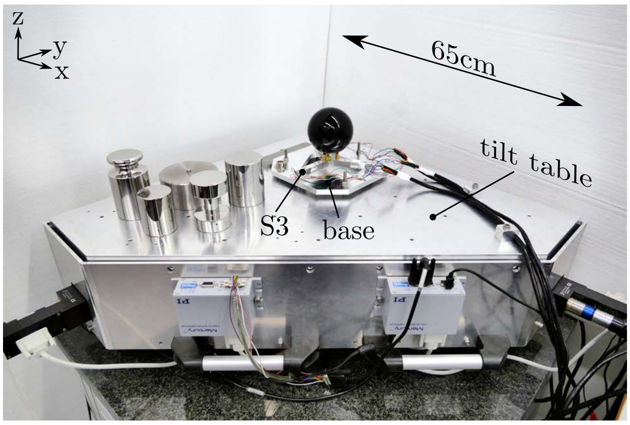

Figure 2 Experimental setup with different mass samples placed on the high precision tilt table. The center of gravity measuring device is equipped with the glass sphere on the three-point support.

during the entire measurement process. Therefore, the mass artifacts are placed on three load cells using pins with hemispherical tips (see figures 1, 2, 3 and 4). The load cells are arranged in 120°, while the pins are arranged with a pitch radius r_P of 15 mm. The three arranged load cells are attached to a base located on a high-precision tilt table with a tilt repeatability of less than 0.4 µrad [2]. Carefully tilting the tilting table about two orthogonal axes changes the forces on the pins due to the deflection of the center of gravity, as seen in Figure 3.

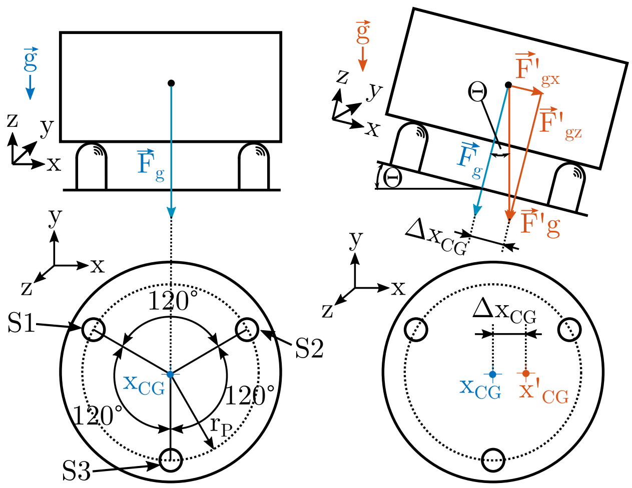

Figure 3 Shift of the force application due to tilt and geometric arrangement of the pins under the sample masses.

The figure shows two states, the initial state where $\vec{g}$ is parallel to the z-axis (left) and the tilted state where $\vec{g}'$ is deflected by Θ relative to the mass sample (right). The load cells can only measure the force component in measuring direction along the z-axis, for the tilted case $\overrightarrow{F_{gz}}$. Using the Equations 1 and the following, the eccentricity of the load on the pins can be calculated. In the tilted state, the force application point has shifted on the x-axis by Δx_{CG} according to Figure 3. Similar to the observation in 2D,

ISBN 978-3-8007-5835-7

the eccentricity can be calculated according to Equations 1 to 9.
The influence of the initial eccentricity is not obstructive for the measurements since only the change of the forces application point relative to the load cells over a defined angle is necessary to determine the center of gravity in z-axis. The Equations 1 and 2 are modified for the relative force change in Equations 6 and 7 to determine the relative motion of the center of gravity to the three pins due to tilt (see Equation 9 and Figure 3).

$$x_{\mathrm{CG}} = r_{\mathrm{P}} \cdot \sin(60°) \cdot (F_{\mathrm{S2}} - F_{\mathrm{S1}}) \cdot F_{\mathrm{g}}^{-1} \tag{1}$$

$$y_{\mathrm{CG}} = r_{\mathrm{P}} \cdot (\sin(30°) \cdot (F_{\mathrm{S1}} + F_{\mathrm{S2}}) - F_{\mathrm{S3}}) \cdot F_{\mathrm{g}}^{-1} \tag{2}$$

with:

$$F_{\mathrm{g}} = \sum_{1}^{3} F_{\mathrm{S}i} \tag{3}$$

Or described in polar coordinates:

$$r_{\mathrm{CG}} = \sqrt{x_{\mathrm{CG}}^2 + y_{\mathrm{CG}}^2} \tag{4}$$

$$\varphi_{\mathrm{CG}} = \arctan\left(\frac{y_{\mathrm{CG}}}{x_{\mathrm{CG}}}\right) \tag{5}$$

$$\Delta x_{\mathrm{CG}} = r_{\mathrm{P}} \cdot \sin(60°) \cdot (\Delta F_{\mathrm{S2}} - \Delta F_{\mathrm{S1}}) \cdot F_{\mathrm{g}}^{-1} \tag{6}$$

$$\Delta y_{\mathrm{CG}} = r_{\mathrm{P}} \cdot (\sin(30°) \cdot (\Delta F_{\mathrm{S1}} + \Delta F_{\mathrm{S2}}) - \Delta F_{\mathrm{S3}}) \cdot F_{\mathrm{g}}^{-1} \tag{7}$$

$$\Delta r_{\mathrm{CG}} = \sqrt{\Delta x_{\mathrm{CG}}^2 + \Delta y_{\mathrm{CG}}^2} \tag{8}$$

$$z_{\mathrm{CG}} = \frac{\Delta r_{\mathrm{CG}}}{\sin(\Delta\Theta)} \tag{9}$$

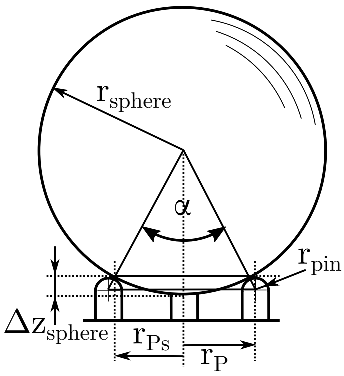

Figure 4 Geometric change of pitch diameter by measuring spheres.

When the center of gravity is measured, the result always refers to the pin tips. For mass samples with flat bottom, no additional considerations are necessary, but when it comes to measuring the center of gravity of a sphere, the pitch diameter of the contact points with the pins changes (see Figure 4).
If the weighing pan of the vacuum mass comparator uses the same pitch circle diameter for a three-pin support, the results are directly transferable. In a cross-check with the measured values from the sphere diameter and the measurement results, the measured center of gravity for the sphere must be adjusted with $z_{\mathrm{CG}} = h_{\mathrm{meas}} + \Delta z_{\mathrm{sphere}}$ according to Equation 12. By applying the first cosine theorem in Euclidean geometry Equation 10, the pitch radius for the sphere r_{Ps} from Figure 4 can be calculated according to Equation 11. Only with the new pitch radius the calculation of the height of the geometrical center of gravity of the sphere can be calculated correctly according to Equation 6 and Equation 7.

$$\alpha = \arccos\left(\frac{(2r_{\mathrm{P}})^2 - 2(r_{\mathrm{sphere}} + r_{\mathrm{pin}})^2}{-2(r_{\mathrm{sphere}} + r_{\mathrm{pin}})^2}\right) \tag{10}$$

$$r_{\mathrm{Ps}} = r_{\mathrm{sphere}} \cdot \sin(\alpha/2) \tag{11}$$

$$\Delta z_{\mathrm{sphere}} = r_{\mathrm{sphere}} - r_{\mathrm{sphere}} \cdot \cos(\alpha/2) \tag{12}$$

4 Experiment and results

Before measurements can begin, the glass sphere is used to calibrate the device. The sphere is self-centering in the three-point support (see Figure 4), ensuring that the mass acting in the three pins is equal to $1/3$ of the weight of the sphere m_{sphere}. Calibration is performed by ABBA comparison of the unloaded and the loaded device. The offset voltages of the load cells and the calibration factors for the calculations are determined. The weight of the glass sphere was determined by comparison with a 1 kg E2 mass standard. To investigate the geometric center of gravity of the sphere, the diameter was determined at various points on the surface using a tactile Abbe comparator (see Table 1).

Property	measured value	std.-dev.
m_{sphere}	925.189 g	±0.127 g
d_{sphere}	88.0341 mm	±7.16 µm
$\vec{g}$	$9.81015772\,\mathrm{m\,s^{-2}}$	$\pm 1.1 \times 10^{-7}\,\mathrm{m\,s^{-2}}$
$\frac{\mathrm{d}\vec{g}}{\mathrm{d}h}$	$-3.153 \times 10^{-6}\,\mathrm{m\,s^{-2}}$	$\pm 1 \times 10^{-8}\,\mathrm{m\,s^{-2}}$

Table 1 Absolute and relative gravitational acceleration at the measurement site and measured properties of the glass sphere.

The main experiments were performed for six weight normals with different weights and shapes (see Figure 6 and Table 2). The tilt table was prepared to be moved to nine positions and wait 5 min during the data recording according to the predefined angles in mrad:

$$\begin{pmatrix}\Theta\\ \Phi\end{pmatrix} = \begin{pmatrix}0 & 15 & 0 & -15 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & -15 & 0\end{pmatrix}$$

The relative force variations indicated by the tilt of the mass sample 4 are shown in Figure 5. As an example, the relative force variation can be clearly observed. For each mass sample from Figure 6, at least three measurement routines were performed and the samples were replaced by hand after each measurement routine. Using the above equations, the geometric height of the center of mass for each sample was calculated and compared to the measured geometric shape. The results are presented in Table 3. The theoretical values differ less than 1 % from the calculated results from the measured data.

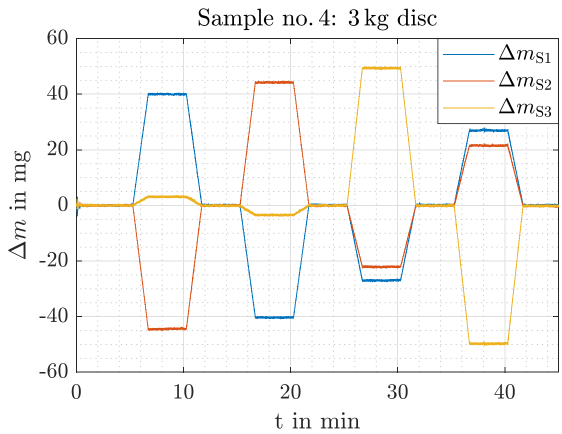

Figure 5 Sample raw data from one measurement with sample 4.

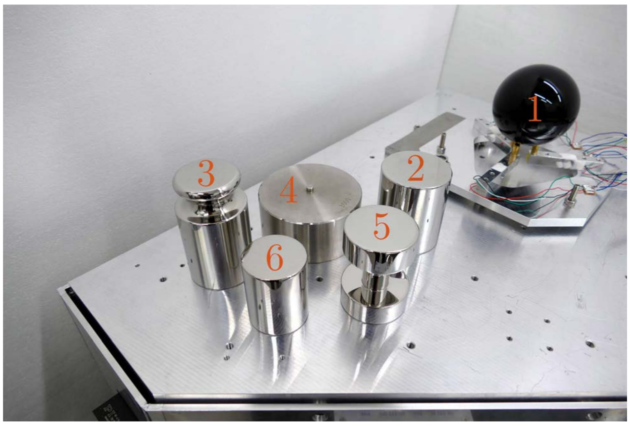

Figure 6 Different mass normals used to check the performance of the new device.

No.	sample	h_{geom} in mm
1	0.925 kg glass sphere	44.02
2	2 kg steel cylinder	39.15
3	2 kg button weight (E2)	46.40
4	3 kg steel disc	24.50
5	1 kg dumbbell weight	40.50
6	1 kg steel cylinder (F1)	32.00

Table 2 Mass samples for the measurements.

#	h_{geom} in mm	h_{meas} in mm	std.-dev. in mm	rel. dev. in %	abs. dev. in μm
1	44.02	44.168	0.024	0.34	148
2	39.15	39.279	0.116	0.33	129
3	46.40	46.758	0.061	0.77	358
4	24.50	24.518	0.054	0.07	18
5	40.50	40.751	0.123	0.62	251
6	32.00	31.738	0.142	−0.82	−262

Table 3 Geometrical center of gravity compared to the measured height of the center of gravity. Standard deviation for $n = 3$ and $k = 1$.

5 Conclusion

The new center of gravity meter showed very good agreement with the geometrically determined center of gravity heights and provided promising results. With a standard deviation of the center of gravity height of 142 µm for the 1 kg F1 steel cylinder, the uncertainty contribution due to unknown center of gravity heights of mass samples is reduced to $< 4.5 \times 10^{-11}$. For further development, it is conceivable to mount actuators on the common base of the three load cells that can tilt the base. This would save space and costs and the device can be used independently. However, it should be noted that the tilt actuators must be calibrated or the tilt must be measured with an independent inclinometer.

6 Literatur

[1] Michael Borys, Roman Schwartz, Arthur Reichmuth, and Roland Nater. Fundamentals of mass determination, 2012.

[2] M Rivero, M Kühnel, and T Fröhlich. High precision dual axis tilt stage. In *Proceedings of the 58th Ilmenau Scientific Colloquium*, 2014.

[3] Christoph Reigber and Peter Schwintzer. Das schwerefeld der erde: Spezial: Geophysik. 34:206–212, 2003. ISSN 0031-9252. doi: 10.1002/piuz.200301023.

[4] T. J. Quinn. The kilogram: the present state of our knowledge. 40:81–85, 1991. ISSN 0018-9456. doi: 10.1109/tim.1990.1032888.

[5] Michael Stock, Richard Davis, Estefanía de Mirandés, and Martin J. T. Milton. The revision of the si—the result of three decades of progress in metrology. 56: 022001, 2019. ISSN 0026-1394. doi: 10.1088/1681-7575/ab0013.

[6] Selim Solmaz, Mehmet Akar, and Robert Shorten. Online center of gravity estimation in automotive vehicles using multiple models and switching. In *2006 9th International Conference on Control, Automation, Robotics and Vision*, pages 1–7, 2006. doi: 10.1109/ICARCV.2006.345080.

[7] Yoshitaka Mikata, Masami Yamanaka, Kouichi Kameoka, Akinari Okunosono, and Toshiaki Kinoshita. Measuring the center of gravity with truck scale. In *SICE Annual Conference 2011*, pages 405–410. IEEE, 2011.

[8] Edward D. Lemaire, Mario Lamontagne, Hugh W. Barclay, Thomas John, and Guy Martel. A technique for the determination of center of gravity and rolling resistance for tilt-seat wheelchairs. 28:51, 1991. ISSN 0748-7711. doi: 10.1682/jrrd.1991.07.0051.

[9] OIML R 111-1 Edition 2004 (E). online. URL `www.oiml.org/en/files/pdf_r/r111-1-e04.pdf`.